Seismic Activity Rates in the Iberian Peninsula

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LISBOA 2012

SUMMARY:
Evaluating the seismic hazard requires establishing a distribution of the seismic activity rate, irrespective of the methodology used in the evaluation. In practice, how that activity rate is established tends to be the main difference between the various evaluation methods.
The traditional procedure relies on a seismogenic zonation and the Gutenberg-Richter (GR) hypothesis. Competing zonations are often compared looking only at the geometry of the zones, but the resulting activity rate is affected by both geometry and the values assigned to the GR parameters. Contour plots can be used for conducting more meaningful comparisons, providing the GR parameters are suitably normalised.
More recent approaches for establishing the seismic activity rate forego the use of zones and GR statistics and special attention is paid here to such procedures. The paper presents comparisons between the local activity rates that result for the complete Iberian Peninsula using kernel estimators as well as two seismogenic zonations.
It is concluded that the smooth variation of the seismic activity rate produced by zoneless methods is more realistic than the stepwise changes associated with zoned approaches; moreover, the choice of zonation often has a stronger influence on the results than its fairly subjective origin would warrant. It is also observed that the activity rate derived from the kernel approach, related with the GR parameter "a", is qualitatively consistent with the epicentres in the catalogue. Finally, when comparing alternative zonations it is not just their geometry but the distribution of activity rate that should be compared.

Keywords: seismic activity rate, kernel estimator, Iberian Peninsula

1. INTRODUCTION

The tasks involved in a Probabilistic Seismic Hazard Analysis (PSHA) can be grouped as follows:

- Compilation of the seismic catalogue, including homogenisation, declustering, identification of uncertainties, and study of its completeness.
- Construction of the seismic activity rate, which represents the number of expected events per year, as a function of magnitude and location.
- Selection of an adequate attenuation model.
- Combination of the seismic activity rate with the attenuation model in order to obtain the relation between ground motion and probability.

The way in which the seismic activity rate is established tends to be the main difference between the various evaluation methods. In the traditional procedure, initially proposed by Cornell (1968), the seismic activity rate is assumed to have a stepwise dependence on location: the activity is considered to be constant over each seismogenic zone, with an exponential dependence on magnitude following the well known Gutenberg-Richter (GR) relation.

The delineation of zones involves a subjective component, which is particularly significant in areas like the Iberian Peninsula, where there is no clear association between earthquakes and seismic sources. Together with the high uncertainty associated with the characterisation of many of the historical events, this leads to very different representations of the seismic activity rate for the same area. For the Iberian Peninsula, the work by Giner et al (2002) presents five zonations used by different authors for the South-East of Spain, each of them leading to considerably different seismic hazard results.

Several ideas have been proposed for avoiding, or at least mitigating, this drawback. For example, Bender (1986) employed the uncertainties in epicentral locations to smooth the discontinuities arising at seismogenic zone boundaries; Veneciano and Pais (1986) developed a methodology for automatic generation of the zones based only on the events in the seismic catalogue.

Frankel (1995) divided the space into bins, assigning values to the *a* parameter of the GR relation for each of these bins and a regional *b* value. In this manner the activity rate presents a histogram-like variation with respect to location, while the dependence on magnitude follows the exponential form of the GR relation.

Woo (1996) also proposed a methodology, inspired in the Non-Parametric Density Estimation (NPDE). The seismic activity rate is constructed with kernel functions placed at the epicentres of the events in the seismic catalogue. The activity rate is not forced to follow any predefined dependence with respect to location or magnitude, rather the events in the catalogue will configure by themselves the shape of the activity rate function.

All three methodologies have now been employed to investigate the seismic hazard in the Iberian Peninsula. Mezcua et al (2011) used the zoned approach by Cornell (1968), as was also done for developing the seismic hazard map of the current Spanish Seismic Code NCSE-02 (Ministerio de Fomento, 2003). The histogram procedure of Frankel (1995) was applied by Peláez (2002). And, very recently, Crespo (2011) conducted the calculations using the kernel methodology by Woo (1996). The purpose of the present study is to analyse how the activity rate is modelled with the kernel methodology and to compare it with the rates employed in other approaches.

2. METHODOLOGY

The methodology employed here follows a Non-Parametric Density Estimation (NPDE) and, more specifically, that originally proposed by Woo (1996).

In NPDE, the objective is to find the density function from which a given sample derives without imposing a priori a specific shape for the density function; instead, the shape of the distribution is expected to be provided by the sample itself.

The method consists in centring a density function on each element of the sample, adding all such functions, and finally normalising their sum. It was initially proposed by Fix and Hodges (1951) and a very clear description is provided by Silverman (1986). The application of NPDE to seismic data was suggested by Vere-Jones (1992). Later on, Woo (1996) presented a way of using kernel functions for modelling seismic activity rates in seismic hazard evaluations. The mathematical definition of the density estimated with kernel functions is as follows:

$$f_n(\mathbf{x}) = \frac{1}{nH^2} \sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{H}\right) \tag{1}$$

where: n is the number of elements in the sample
H is the bandwidth, which is a measure of the separation between sample elements
K is the kernel function
$\mathbf{x}_i$ is the position of event i

For generating a seismic activity rate density λ_k, two changes are introduced in the above expression:

- The normalisation with respect to the number of events n is omitted, thus the result is expressed in terms of number of events.
- Each kernel function is divided by an effective period of detection T, hence the density of events is expressed per unit time.

With the above changes, the density becomes:

$$\lambda_k(M,\mathbf{x}) = \frac{1}{[H(M)]^2} \sum_{i=1}^{n} \frac{K\left(\dfrac{\mathbf{x}-\mathbf{x}_i}{H}\right)}{T(\mathbf{x}_i)} \tag{2}$$

The resulting activity rate density λ_k, depends on location as well as magnitude through the bandwidth H. As can be seen, it is a summation of kernel functions K, placed on each event of the catalogue with coordinates $\mathbf{x}_i$. Each function is weighted with an effective detection period T which is a measure of the detection probability of that event in past times; the normalisation is achieved through the bandwidth H which depends on the distance between events. The kernel function K, the effective detection period T, and the bandwidth H are the three main parameters that influence the activity rate.

Several kernel functions have been proposed when the sample is of seismic type, specifically the Gaussian kernel, the Inverse Bi-Quadratic (IBQ) kernel, and a finite one that vanishes for distances beyond one bandwidth. The first two kernels were already suggested by Vere-Jones (1992); Woo (1996) proposed employing the IBQ kernel and, alternatively, also a finite one. All three types of kernels can be seen in Figure 1. The IBQ kernel is the one used here, which is expressed as follows:

$$K_{IBQ}(\mathbf{x}) = \left(\frac{\lambda_{IBQ}-1}{\pi}\right)\left(1+\mathbf{x}^T\mathbf{x}\right)^{-\lambda_{IBQ}} \tag{3}$$

where: λ_{IBQ} is greater than 1 for which Vere-Jones (1992) suggests values between 1.5 and 2.0.

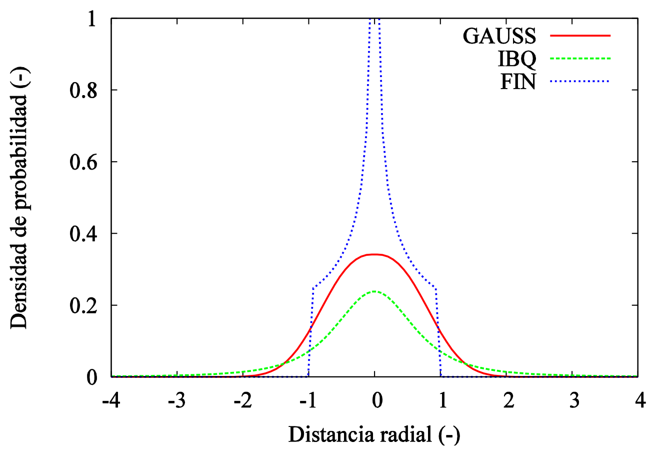


Figure 1. Kernel functions proposed so far for seismic activity representation

The effective period of detection T must be defined for every earthquake in the catalogue. If the catalogue were complete, this effective period of detection would be the period of time covered by the events; since in most cases, including the present one, this is not the case, effective periods of detection have to be established. The purpose is to capture correctly the temporal activity rate without eliminating any event, since this would affect the spatial distribution of the activity rate density λ_k.

The bandwidth H is assumed to depend on magnitude through an exponential law (Woo, 1996):

$$H(M) = c \exp(dM) \tag{4}$$

where: c and d are parameters to be fitted using the catalogue data
M is the magnitude, or the measure employed for the quantification of event size

For the purposes of this study, the accumulated activity rate density λ^*_k (M, $\mathbf{x}$) will be calculated, which represents the events of magnitude greater than or equal to M, per year and unit area:

$$\lambda_k^*(M, \mathbf{x}) = \int_M^{M_{\max}} \lambda_k(m, \mathbf{x}) \mathrm{d}m \tag{5}$$

It should be noted that, strictly speaking, there is no need to calculate the accumulated activity rate $\lambda^*_k(M, \mathbf{x})$ for determining the seismic hazard, but this representation of seismic activity allows more straightforward comparisons with the traditional zoned procedures.

3. SEISMIC CATALOGUE

The main catalogue used in the study is the seismic database maintained by the Spanish Instituto Geográfico Nacional (IGN). For some areas this catalogue has been supplemented with data from two international organisations (USGS and ISC), data from the catalogues of neighbouring countries (BRGM, 2010; Gruppo di Lavoro CPTI, 2004), and other published works that describe the seismicity of surrounding regions (Vilanova and Fonseca, 2007; Peláez et al, 2007).

The quantification of events has to be homogeneous throughout the catalogue. The homogenisation has been conducted in terms of moment magnitude M_w. Some 10% of the events in the IGN catalogue are only quantified with epicentral intensity, as is frequently the case for events from the historical period. During the instrumental period most events have an assigned magnitude; generally this magnitude is either m_b or m_{bLg} and, since 2002, some of them have also a moment magnitude M_w. The conversion of both epicentral intensity and magnitude m_b or m_{bLg} into moment magnitude M_w has been carried out using correlations specifically developed for the Iberian Peninsula by the IGN and already employed by Crespo (2011). The moment magnitudes assigned by the IGN have been maintained, and other moment magnitudes from the literature have also been incorporated (Stich et al, 2003, 2010).

It is assumed that earthquake events are Poisson-distributed, which requires removing dependent events. The methodology followed for this purpose is the traditional one of placing a time-space window around the main events to identify those that need to be discarded. The procedure is similar to that proposed by Gardner and Knopoff (1974), modified here using appropriate parameters for the seismicity of the Iberian Peninsula to ensure that the main event is the first one identified in each iteration; a detailed description is offered by Crespo (2011). As a result of the pruning process conducted, 36% of the events were considered dependent and discarded from the database.

The methodology used requires the catalogue to incorporate uncertainties with respect to magnitude as well as to location (epicentral and depth). Regarding magnitude, the uncertainties quoted in the various catalogues have been taken into account. When using a correlation, its uncertainties have been combined with those in the catalogue.

As for the location uncertainty, the catalogues provide this type of information for many earthquakes, especially in the instrumental period. For events lacking this information in the catalogue appropriate uncertainties have been assigned.

4. RESULTS

Figure 2 presents the seismic activity rate obtained for the Iberian Peninsula following the methodology indicated in section 2. Specifically, the figure shows the annual number of events per km^2 with magnitude M_w greater than or equal 3.5; it can be noticed that apart from being expressed per unit area, this activity rate is equivalent the GR ordinate for M_w = 3.5.

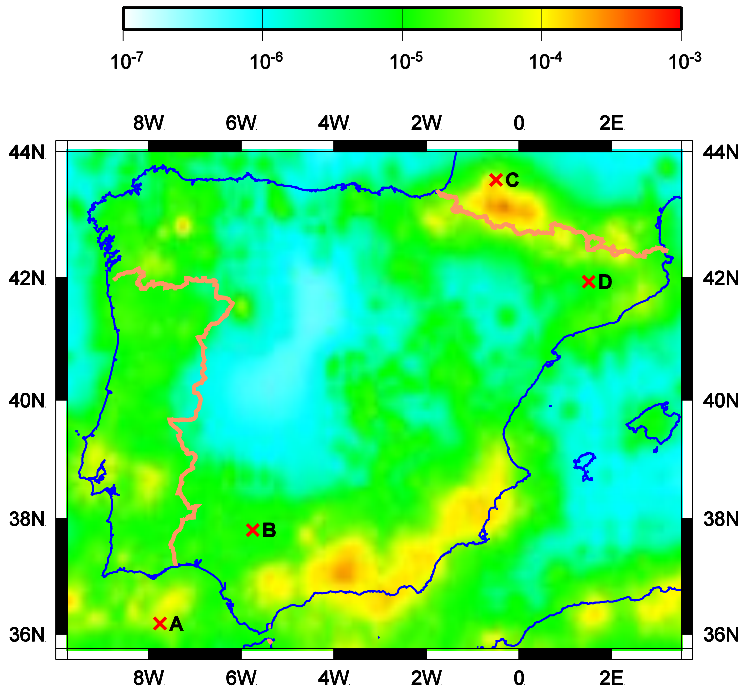


Figure 2. Activity rate for M_w > 3.5 (events/km^2/year) for the Iberian Peninsula with the kernel methodology.

The results plotted are entirely consistent with the known seismicity: the region around Granada and the central Pyrenees, already over the French territory, are those with the higher values. They are also consistent with the comparable findings by Peláez (2000), which are reproduced in Figure 3.

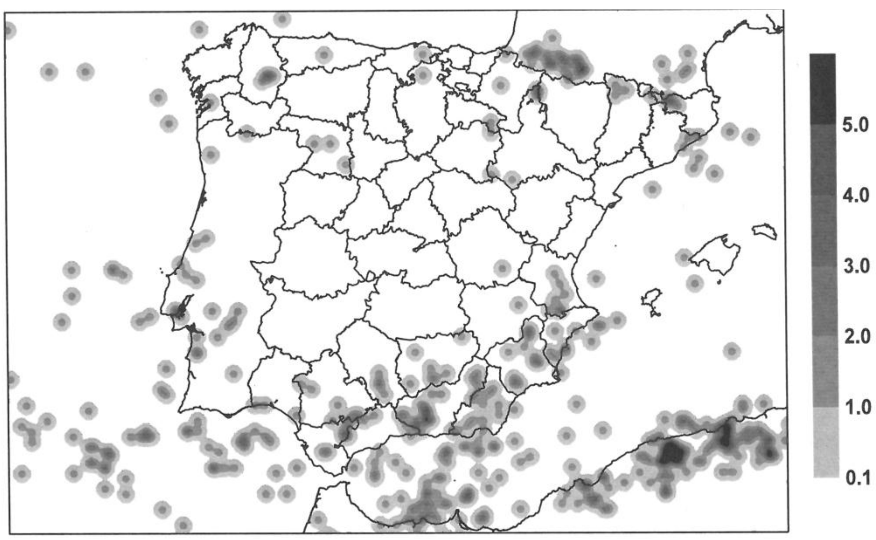
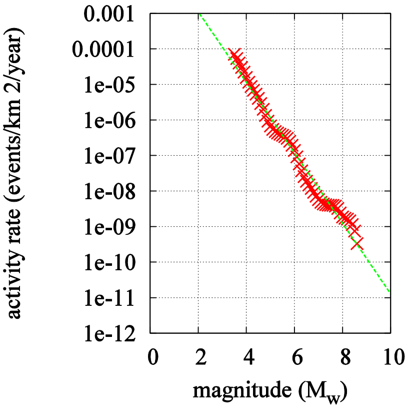
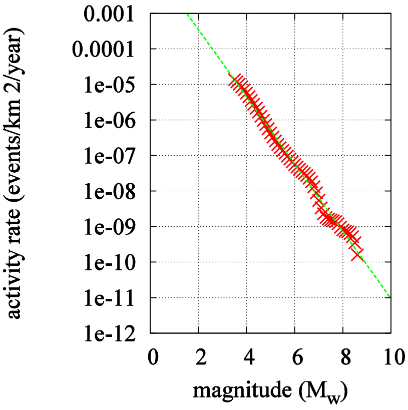


Figure 3. Number of events with M_s > 3.5 per bin of 10 km x 10 km for 100 years (Peláez, 2000).

As indicated in equations (2) and (5), the seismic activity rate depends on both magnitude and location. Figure 2 presents the variation with location for a fixed magnitude (3.5 in this case). That with respect to magnitude is represented in Figure 4 for the four locations highlighted in Figure 2.



Point A



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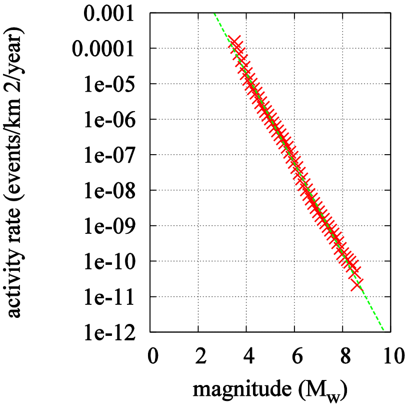

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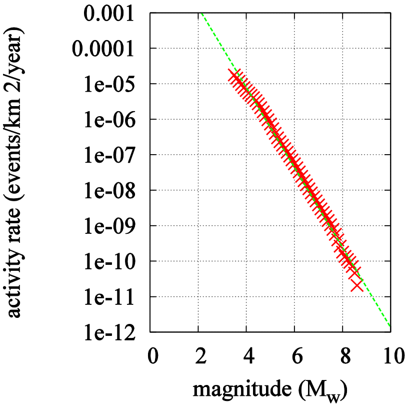

Point D

Figure 4. Activity vs. magnitude for selected locations. Kernel methodology.

The red crosses plotted in Figure 4 represent the activity rate obtained using the kernel estimators. As can be seen the accumulated seismic activity rate decreases with increasing magnitude, as is also the case in the GR relation. Although the relation obtained here is non-linear, a least square linear fit of the results would produce the line shown in green, with a slope b_k. This green line is equivalent to the GR relation used in zoned approaches and, as shown by Crespo (2011), its slope b_k can be directly compared with the b value of the GR relation.

The same exercise was repeated for all points on the map and a b_k value was obtained for each point. The resulting distribution of the b_k values in the Iberian Peninsula is plotted in Figure 5.

The b_k slopes vary smoothly over the entire area of study, with values ranging from 0.8 in the central part of the Peninsula up to 1.3 around Granada and 1.4 in the central Pyrenees. With the exception of these two areas, the values are in agreement with those usually found in PSHA studies, as well as with the ones originally indicated by Gutenberg and Richter (1944):

$$b = 1 \pm 0.2 \tag{3}$$

At the same time, the smooth contours shown in Figure 5 are in clear contrast with the stepwise variations that appear in traditional zoned approaches.

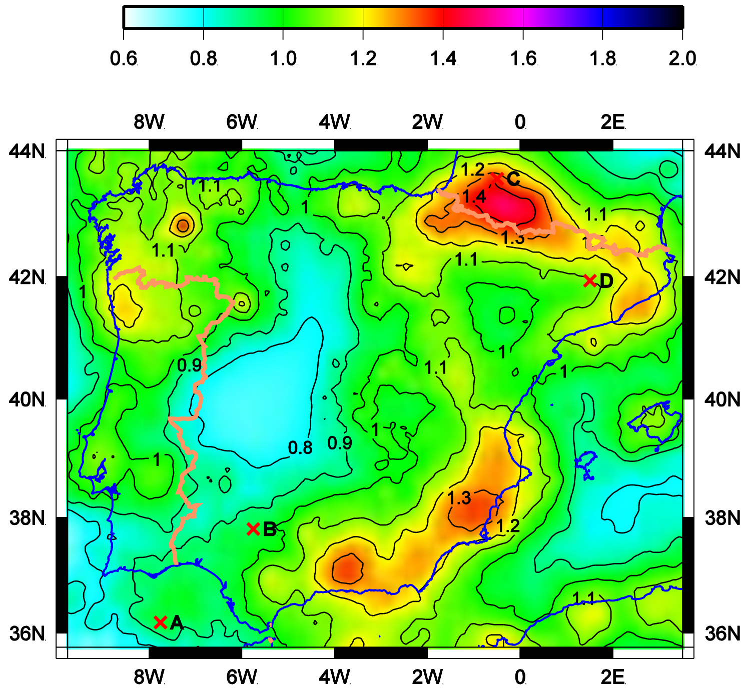

Figure 5. Distribution of b_k values for the Iberian Peninsula using the kernel methodology.

Two zonations have been selected to provide comparisons with the previous findings. The first one, identified as NCSE, is that used for developing the hazard map of the current Spanish seismic code NCSE-02 (Ministerio de Fomento, 2003). The other one, labelled GM, was recently developed by García-Mayordomo (2012) and will be published shortly. The *b* values characterising each zone, derived in the context of a currently ongoing project, were provided by Rivas (2012); although they are not yet final and may still suffer slight variations, they are considered sufficient for the present comparison.

Figure 6 presents the NCSE ordinate for $M_w = 3.5$ according to the GR relation, normalised with the area of the corresponding zone. This value is directly comparable with the activity rate presented in Figure 2. As can be seen, there is a good qualitative and quantitative agreement, considering that the size of the zones does not allow capturing the gradients found in Figure 2.

Figure 7 depicts the distribution of NCSE *b* values, while Figure 8 shows the GM *b* values. In the case of the NCSE, the *b* values are between 0.8 and 1.2, with the average value of 1.0 in most of the zones; in the case of GM, some higher values are present, particularly for the primarily maritime zone located South-East of the Peninsula. In spite of these differences, there is a fair degree of consistency in the results, though the size of the seismogenic zones used in zoned approaches does not allow obtaining the richer details produced by the kernel methodology.

4. CONCLUSIONS

Kernel estimators have been used to construct an activity rate for the complete Iberian Peninsula without having to establish a set of a seismogenic zones. Although not necessary for evaluating the seismic hazard, parameters equivalent to the *a* and *b* parameters of the Gutenberg-Richter relation were derived from the exercise; this allows making comparisons with the activity rates employed when making use of zoned procedures.

The following conclusions can be offered as a result of this work:

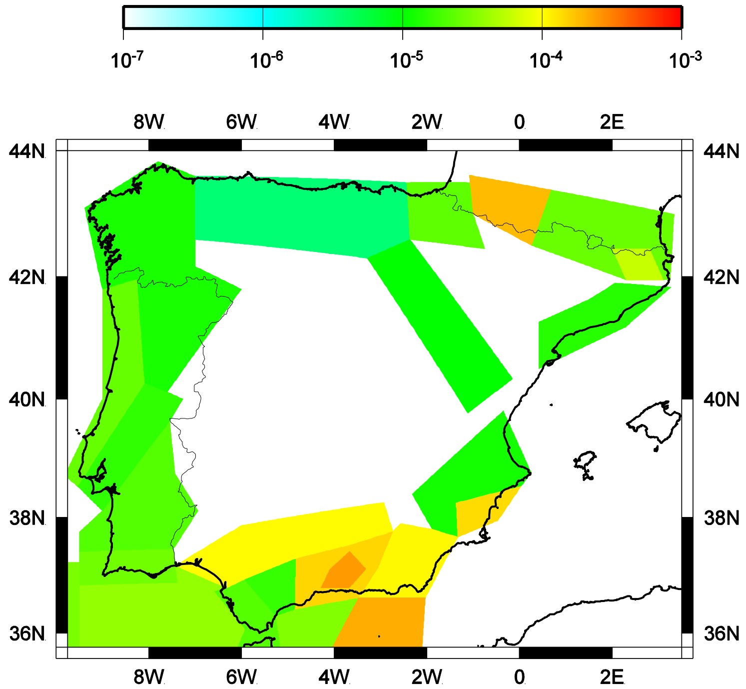

Figure 6. GR value for M_w = 3.5, normalised with zone areas, for the Iberian Peninsula with the NCSE02 zones.

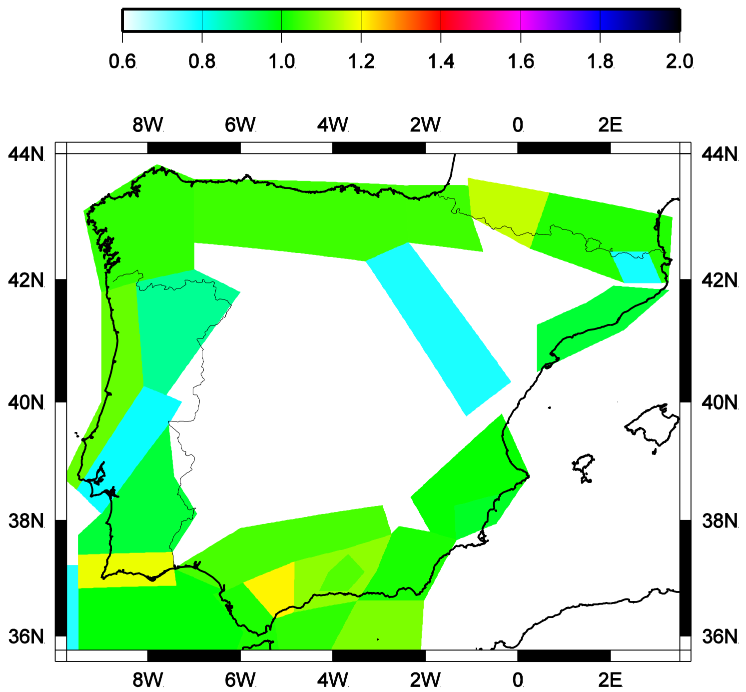

Figure 7. GR *b* value for the Iberian Peninsula with the NCSE02 zones.

a) Woo's zoneless approach based on kernel estimators provides a description of the activity rates in the Iberian Peninsula that is consistent with the known seismicity and includes a rich description of local variations, which are not masked by preconceived ideas as to the geometry of seismogenic zones.

b) Although there is no need to use the equivalent of a Gutenberg-Richter *b* parameter, local values of that parameter can be obtained from the kernel approach. The results are comprised in the range in which this parameter would be expected.

c) The comparison of the results in terms of the b_k parameter, equivalent to the GR *b* one, with those produced in two different zoned approaches by other investigators show that there is reasonable consistency with those results, although the kernel approach allows more precision and a greater degree of detail.

d) Finally, it is worth pointing out that the inevitable subjectivities of zoned approaches are frequently the source of controversy, which are avoided when the activity rates are obtained with kernel estimators or some other zoneless procedure.

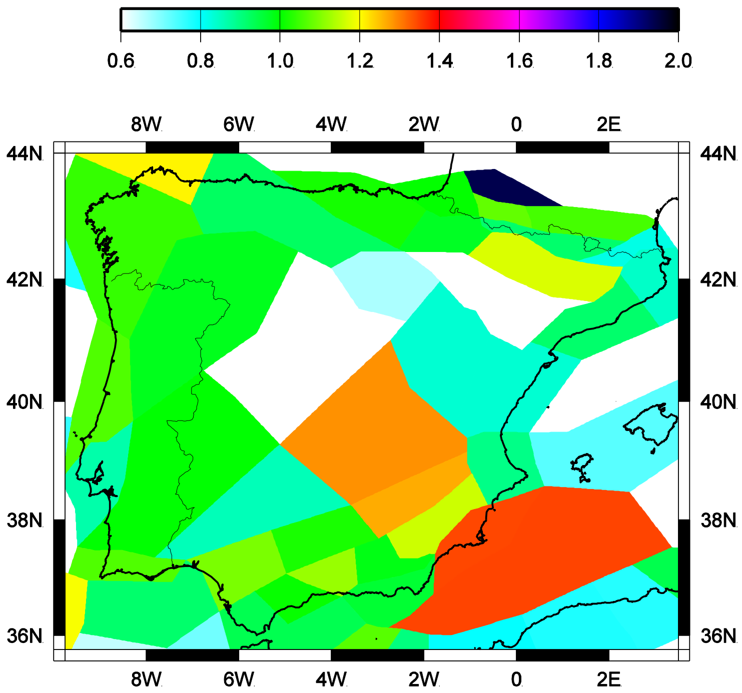

Figure 8. GR *b* value for the Iberian Peninsula with the GM zones.

ACKNOWLEDGMENTS

The authors wish to thank Julián García-Mayordomo for making available his zonation before its publication. They are also grateful to Alicia Rivas, who provided the *a* and *b* parameters of the two zonations mentioned in the paper, based on her work on a currently ongoing research project.

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